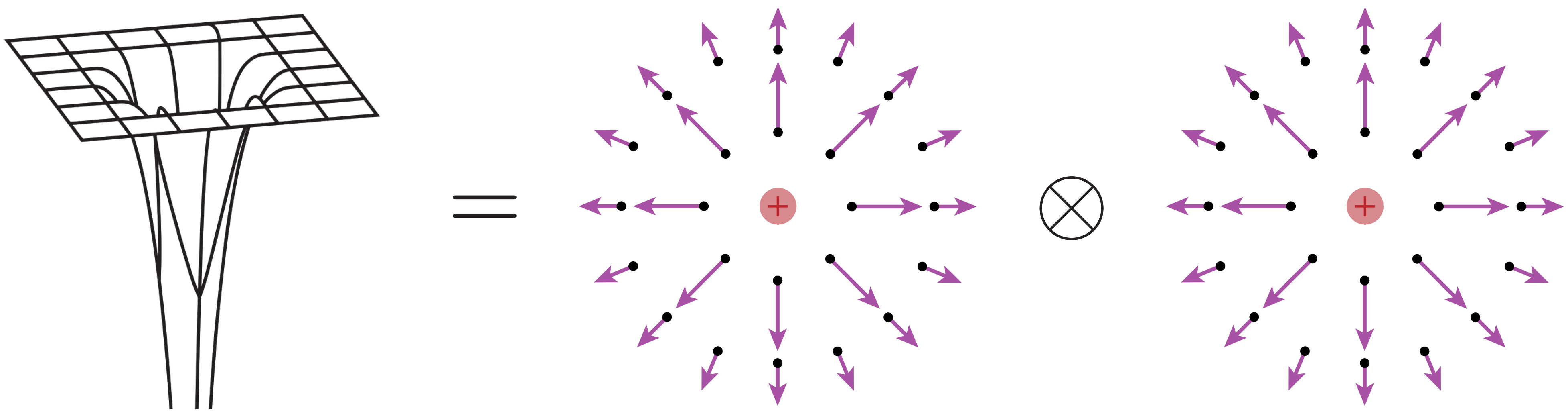
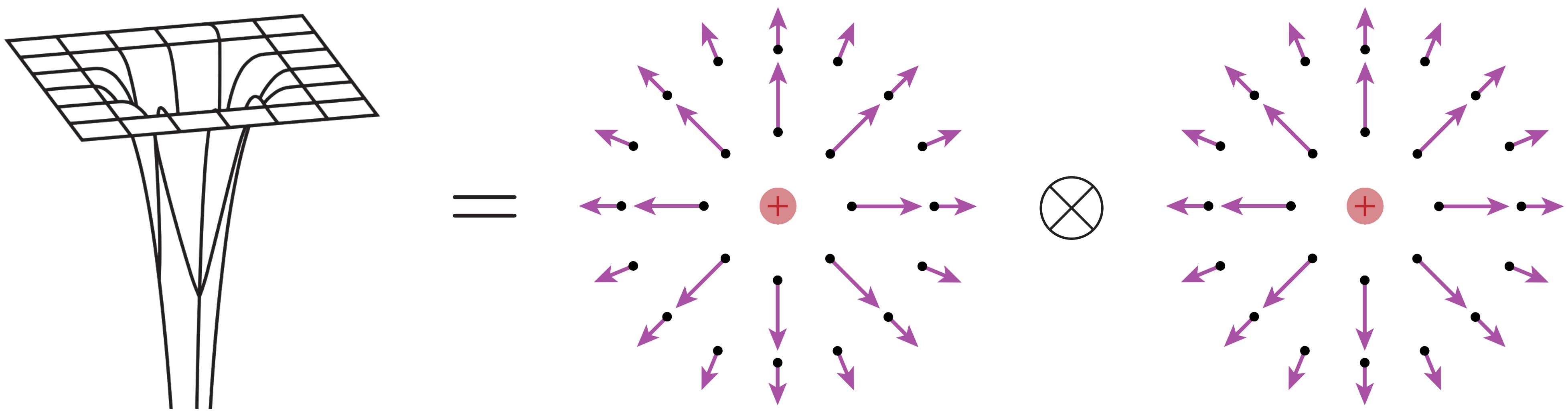


DIAS, April 2020

Quantum Amplitudes and Classical Gravity

Donal O'Connell
University of Edinburgh



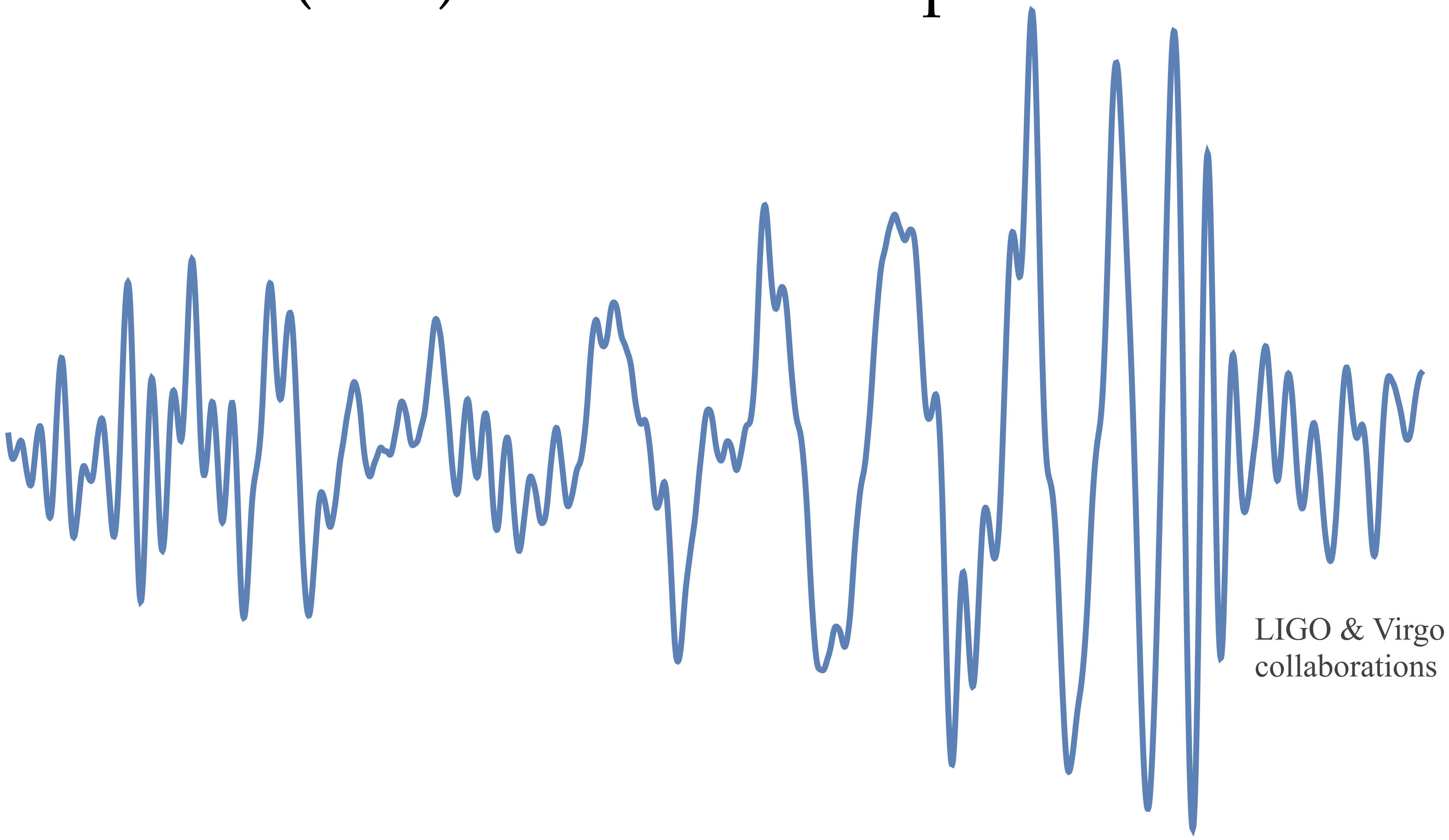
DIAS, April 2020

Quantum Amplitudes and Classical Gravity

Work with Ben Maybee; Uri Kol, Andrés Luna, Justin Vines; Nima Arkani-Hamed, Yu-tin Huang, David Kosower, Ricardo Monteiro, Chris D. White

Golden age for gravity

Gravitational wave (GW) data from compact binaries

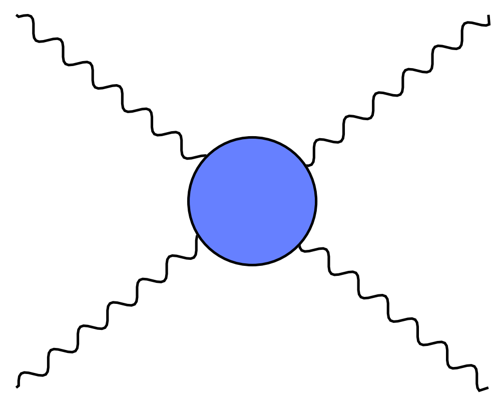


High precision, data rich future!

Golden age for gravity

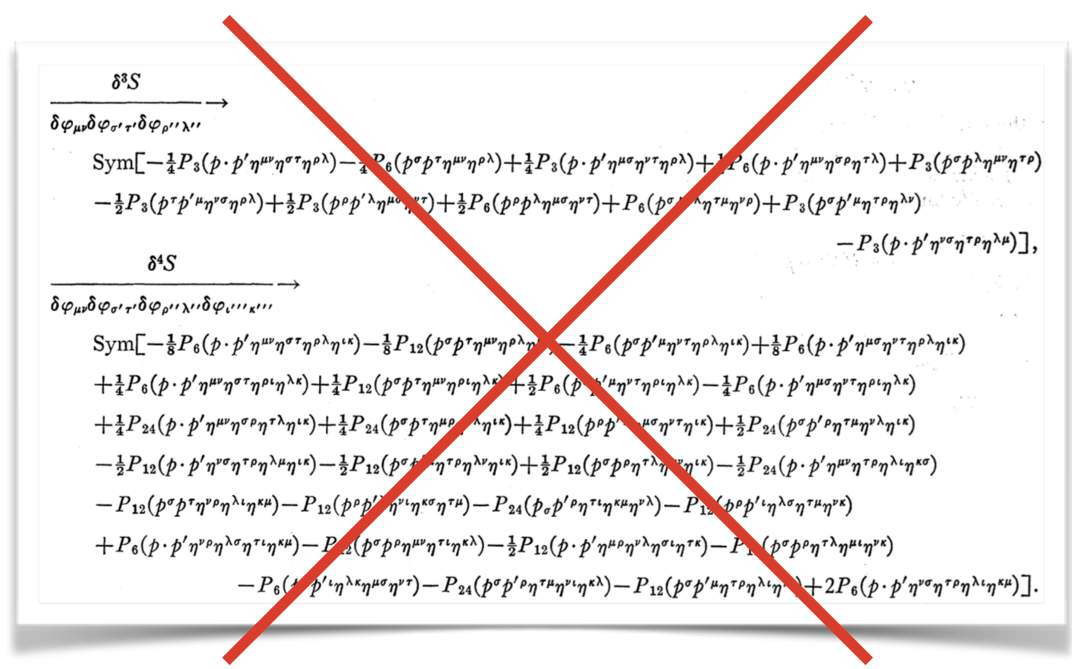
Revolution in quantum scattering in gravity

$$\mathcal{A} = \langle \text{future} | \text{past} \rangle =$$



Unexpected simplicity!

Amplitude much simpler than Feynman rules



$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \longrightarrow$$

$$\text{Sym}[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}) - \tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}) + \tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}) + \tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}) + P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})$$
$$-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda}) + \tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}) + \tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}) + P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho}) + P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})$$
$$-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})],$$

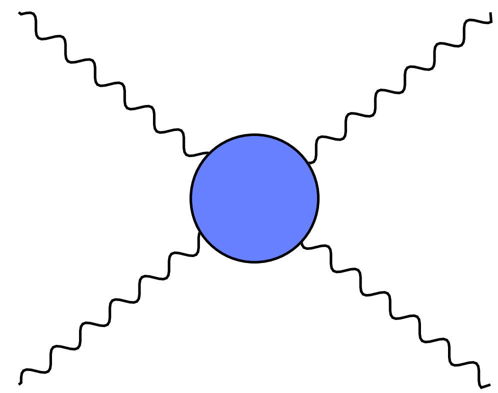
$$\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \longrightarrow$$

$$\text{Sym}[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa}) - \tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa}) - \tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa}) + \tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})$$
$$+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa}) + \tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa}) + \tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa}) - \tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})$$
$$+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa}) + \tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa}) + \tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa}) + \tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})$$
$$-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa}) - \tfrac{1}{2}P_{12}(p^\sigma p^\mu\eta^{\tau\rho}\eta^{\lambda\nu}\eta^{\iota\kappa}) + \tfrac{1}{2}P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\nu}\eta^{\iota\kappa}) - \tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma})$$
$$-P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu}) - P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu}) - P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda}) - P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})$$
$$+P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu}) - P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda}) - \tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\sigma\iota}\eta^{\tau\kappa}) - P_{[illegible]}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa})$$
$$-P_6(p^{[illegible]} p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau}) - P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda}) - P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{[illegible]}) + 2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})].$$

Golden age for gravity

Revolution in quantum scattering in gravity

$$\mathcal{A} = \langle \text{future} | \text{past} \rangle =$$

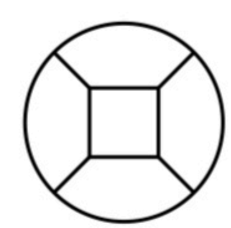


Unexpectedly simple!

Amplitude much simpler than Feynman rules

High-order perturbative calculations possible

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16 \times 629}{25}\left(\frac{\kappa}{2}\right)^{12}(s^2+t^2+u^2)^2 stu M_4^{\text{tree}} \left(\frac{1}{48}\right.$$



$$+\frac{1}{16}$$

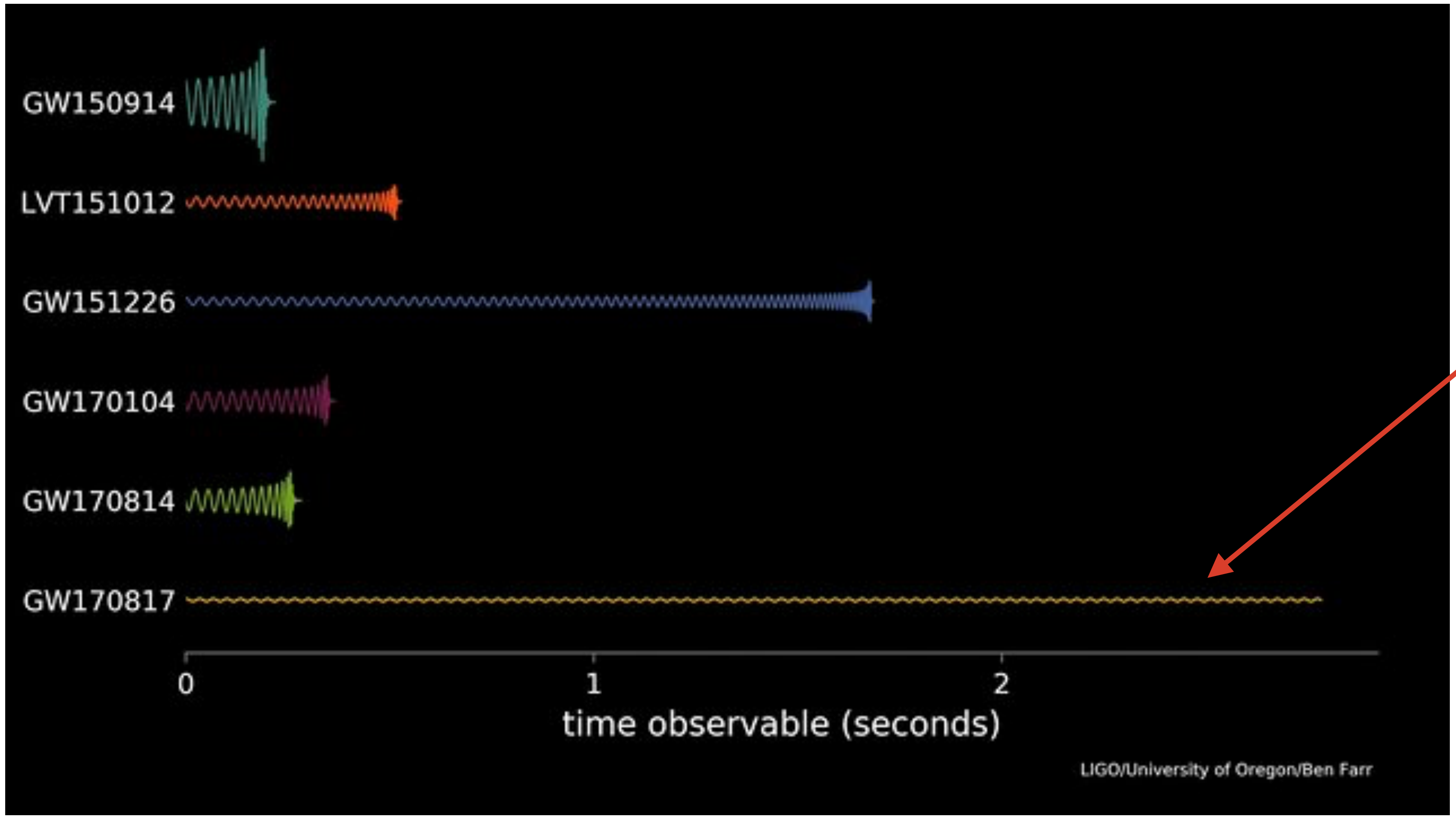
$$\left.\right)$$

$$\kappa^2 = 32\pi G$$

Bern et al, 2018

Golden age for gravity

Gravitational wave data: *perturbative* region information-rich



Binary neutron star
~3000 cycles
~1 minute in band

Eg tidal deformability contributes at order G^6

Golden age for gravity

Progress is in *perturbative* gravity

1. Gravitational wave data: *motivates* classical perturbative gravity
2. Scattering amplitudes: hidden *simplicity* in perturbation theory

Is perturbative classical gravity simpler than it appears?

Outline

1. The double copy
2. Amplitudes & classical scattering
3. Large spin and Kerr
4. Magnetic amplitudes
5. Conclusion

The double copy

"Double copy": for amplitudes, gravity = (Yang-Mills)^2

Kawai, Lewellen, Tye
Bern, Carrasco, Johansson

$$\mathcal{A} = g f^{abc}(\epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot p_1 + \epsilon_2 \cdot \epsilon_3 \epsilon_1 \cdot p_2 + \epsilon_3 \cdot \epsilon_1 \epsilon_2 \cdot p_3) \qquad \text{YM}$$

↕ Straight squares

$$\mathcal{M} = \kappa(\epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot p_1 + \epsilon_2 \cdot \epsilon_3 \epsilon_1 \cdot p_2 + \epsilon_3 \cdot \epsilon_1 \epsilon_2 \cdot p_3)^2 \qquad \text{GR}$$

$$e_1^{\mu\nu} = \epsilon_1^\mu \epsilon_1^\nu$$

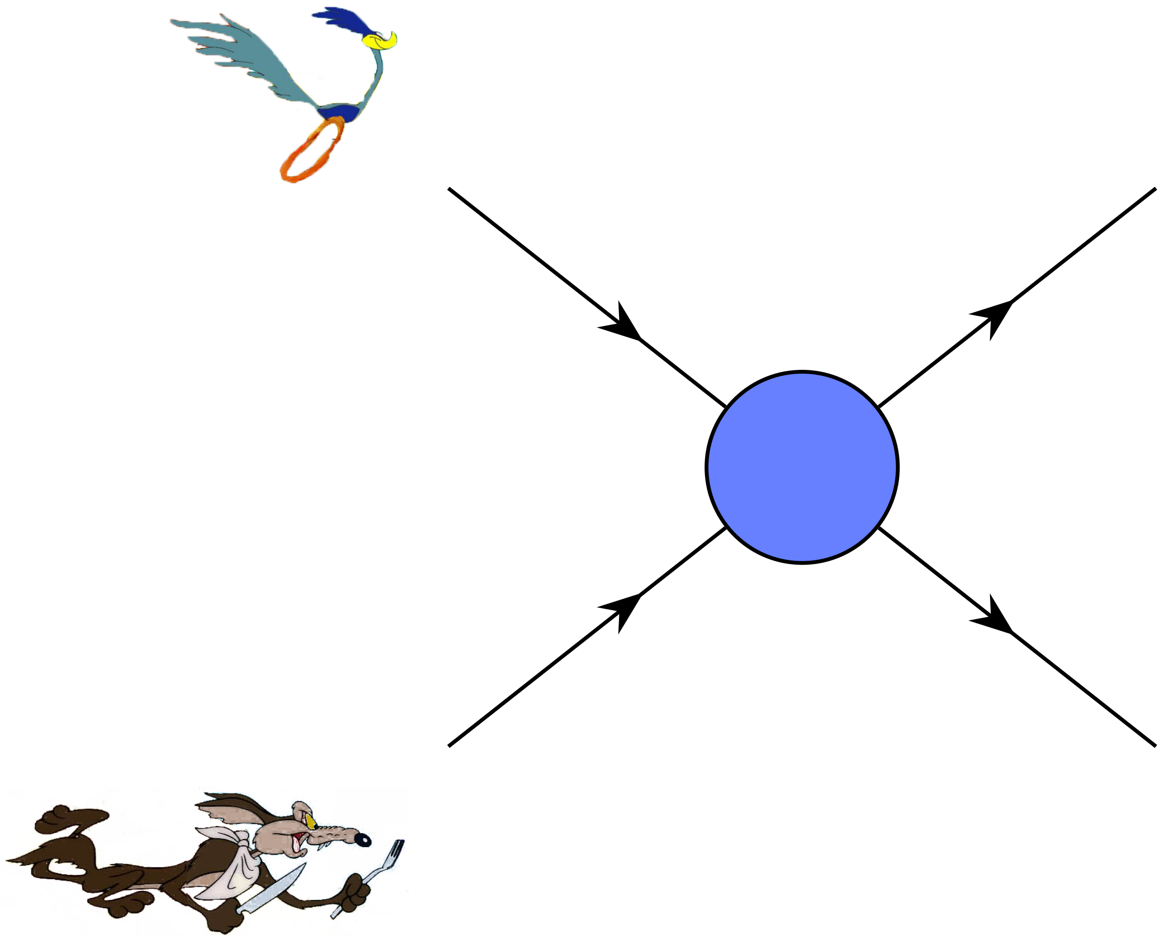
Extends to all orders

Very weird from classical geometric point of view on GR

The double copy

Is there any sign of double copy in classical GR?

Example: consider impulse in classical fast scattering



The double copy

Is there any sign of double copy in classical GR?

Example: consider impulse in classical fast scattering

$$\Delta p_1^\mu = -\frac{e^2}{2\pi}\frac{b^\mu}{b\cdot b}\frac{u_1\cdot u_2}{\sqrt{(u_1\cdot u_2)^2-1}}$$

EM

$$\Delta p_1^\mu = 4Gm_1m_2\frac{b^\mu}{b\cdot b}\frac{(u_1\cdot u_2)^2}{\sqrt{(u_1\cdot u_2)^2-1}}$$

Gravity

Double copy is present in classical GR

Just hasn't really been noticed before

The double copy

Is there any sign of double copy in classical GR?

Example: consider impulse in classical fast scattering

$$\Delta p_1^\mu = -\frac{e^2}{2\pi}\frac{b^\mu}{b\cdot b}\frac{u_1\cdot u_2}{\sqrt{(u_1\cdot u_2)^2-1}}$$

EM

$$\Delta p_1^\mu = 4Gm_1m_2\frac{b^\mu}{b\cdot b}\frac{(u_1\cdot u_2)^2-\frac{1}{2}}{\sqrt{(u_1\cdot u_2)^2-1}}$$

Einstein Gravity

Double copy is present in classical GR

Just hasn't really been noticed before

(Cheating a little)

The double copy

Double copy reflected in structure of exact Kerr-Schild solutions?

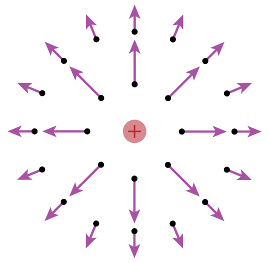
Monteiro, White, DOC

"Classical double copy" proposal

Luna, Monteiro, White, DOC

$$A_\mu = \frac{e}{r} k_\mu$$

Coulomb charge
(embedded in YM)



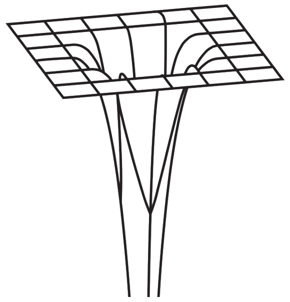
Double copy

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = \frac{2M}{r} k_\mu k_\nu$$

$$k^\mu = (1, \underline{\hat{r}})$$

Schwarzschild
(exact)



The double copy

Double copy reflected in structure of exact Kerr-Schild solutions?

Monteiro, White, DOC

"Classical double copy" proposal

Luna, Monteiro, White, DOC

$$F(x,y,z) = \mathrm{Re}\left[F_{\text{Coulomb}}(x,y,z+ia) + i * F_{\text{Coulomb}}(x,y,z+ia)\right]$$

"$\sqrt{\text{Kerr}}$"

↕ *Double copy*

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = \frac{2M}{r} k_\mu k_\nu$$

Kerr (exact)

The double copy

Double copy reflected in structure of exact Kerr-Schild solutions?

Monteiro, White, DOC

"Classical double copy" proposal

Luna, Monteiro, White, DOC

$$F = \frac{q}{4\pi r^2} dt \wedge dr + \frac{g}{4\pi} \sin\theta \, d\theta \wedge d\phi$$

Dyon

↕ *Double copy*

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$h_{\mu\nu} = \phi k_\mu k_\nu + \psi \ell_\mu \ell_\nu$$

Taub-NUT (exact)

The double copy

Double copy reflected in structure of exact Kerr-Schild solutions?

Monteiro, White, DOC

“Classical double copy” proposal *Luna, Monteiro, White, DOC*

- Lots of evidence for Schwarzschild *Goldberger & Ridgway*

More detailed tests need…

1. Classical observables from amplitudes
2. Amplitudes with large spin
3. Amplitudes with magnetic/NUT charge

Amplitudes & Classical Scattering

Classical scattering

First, show you amplitudes determine some classical observables

- Focus on impulse (total change in momentum during scattering)

$$\Delta p_1^\mu = i \int \hat{d}^4 q \; \hat{\delta}(2p_1 \cdot q)\hat{\delta}(2p_2 \cdot q) \; e^{-ib\cdot q} \; q^\mu \; \mathcal{A}(p_1 p_2 \to p_1 + q, p_2 - q) + \cdots$$

Kosower, Maybee, DOC

$$\hat{d}q \equiv \frac{dq}{2\pi}$$

$$\hat{\delta}(q) \equiv (2\pi)\delta(q)$$

- Sketch the derivation

Classical scattering

- Time evolution operator from far past to far future is the S matrix:

$$U(-\infty, \infty) = S$$

- Write $S = 1 + iT$

Nothing happens

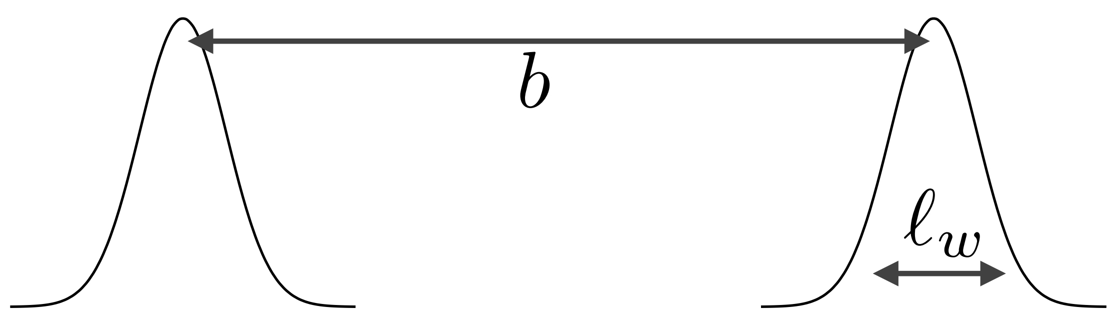
"Transition matrix"

- The matrix elements of T on momentum states are the amplitudes

$$\mathcal{A}(p_1 \ldots p_n \to q_1 \ldots q_m)\delta^4\left(\sum p_i - \sum q_j\right) = \langle q_1 \ldots q_m | T | p_1 \ldots p_n \rangle$$

Classical scattering

- Correspondence principle: quantum = classical + small
  - Valid if quantum effects small compared to classical scales
  - … so put particles in finite-size wavepackets



$$\Delta x = \ell_w \ll b$$

- Don’t worry! Wavepackets disappear in the end
  - Very similar to standard cross-section derivation

Classical scattering

- Correspondence principle: quantum = classical + small
  - Also need spread in momentum space small compared to mass

$$\psi(x) \sim \exp\left[-\frac{x^2}{\ell_w^2}\right] \xrightarrow{\text{Fourier}} \tilde{\psi}(p) \sim \exp\left[-\frac{\ell_w^2 p^2}{\hbar^2}\right]$$

  - Hence $$\Delta p = \frac{\hbar}{\ell_w} \ll m \Rightarrow l_c = \frac{\hbar}{m} \ll \ell_w$$
- Correspondence region when $l_c \ll \ell_w \ll b$
  - Modes in interaction soft compared to modes in wavefunction

Classical scattering

- Scatter distinguishable, stable particles (scalar for now)
- State in the far past:

$$|\psi\rangle_{\text{in}} = \int d\Phi(p_1) d\Phi(p_2)\, \phi_1(p_1)\phi_2(p_2)\, e^{ib\cdot p_1} |p_1\, p_2\rangle_{\text{in}}$$

Particles displaced by impact parameter, *b*

$$d\Phi(p_1) \equiv \frac{d^4 p_1}{(2\pi)^4}(2\pi)\delta(p_1^2 - m_1^2)$$

Integral over massive on-shell phase space

Wavefunctions: peaked at incoming momenta mu^μ.

Classical scattering

- "Impulse" = total change in momentum during scattering

$$\langle p_1^\mu \rangle \equiv \langle \psi | \hat{P}_1^\mu | \psi \rangle \qquad \langle p_1'^\mu \rangle = \langle \psi | S^\dagger \hat{P}_1^\mu S | \psi \rangle$$

Momentum operator

Final state

- Impulse is: $\langle \Delta p_1^\mu \rangle = \langle \psi | S^\dagger \hat{P}_1^\mu S - \hat{P}_1^\mu | \psi \rangle$

$$= \langle \psi | i[\hat{P}_1^\mu, T] | \psi \rangle + \langle \psi | T^\dagger [\hat{P}_1^\mu, T] | \psi \rangle$$

Kosower, Maybee, DOC

- All-orders formula: well-defined classical & quantum observable

Classical scattering

- Lowest order here

On-shell states, momentum transfer negligible in wavefunctions

Wavenumber $\bar{q}$ defined by $q = \hbar\bar{q}$

$$\langle \Delta p_1^\mu \rangle = \langle \psi | i[\hat{P}_1^\mu, T] | \psi \rangle$$

$$\to i \int \hat{d}^4\bar{q} \, \frac{\hat{\delta}(u_1 \cdot \bar{q})}{2m_1} \frac{\hat{\delta}(u_2 \cdot \bar{q})}{2m_2} \, e^{-ib\cdot\bar{q}} \, \bar{q}^\mu \, \mathcal{A}(p_1 p_2 \to p_1 + q, p_2 - q)$$

Momentum transfer

Four-point scattering amplitude

Classical scattering

- So

$$\Delta p_1^\mu = i \int \hat{d}^4\bar{q}\, \frac{\hat{\delta}(\bar{q}\cdot u_1)}{2m_1} \frac{\hat{\delta}(\bar{q}\cdot u_2)}{2m_2}\, e^{-ib\cdot\bar{q}}\, \bar{q}^\mu\, \mathcal{A}(p_1 p_2 \to p_1 + q, p_2 - q)$$

- Useful to note that contact interaction classically irrelevant
  - Contact: $\mathcal{A}(p_1 p_2 \to p_1 + q, p_2 - q) = \text{constant}$
  - But

$$\int \hat{d}^4\bar{q}\, \hat{\delta}(\bar{q}\cdot u_1)\hat{\delta}(\bar{q}\cdot u_2)\, e^{-ib\cdot\bar{q}} = \hat{\delta}^2(b)$$

Outside domain of validity

Large spins: Kerr

Large spins

Arkani-Hamed, Huang, Huang

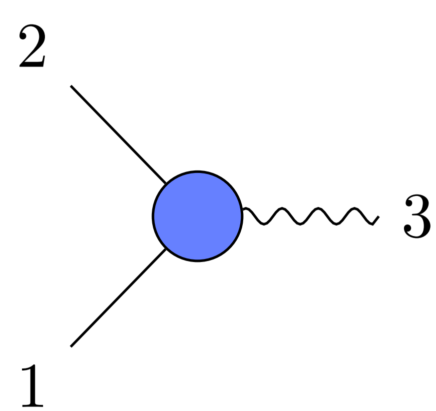
- Recent progress on amplitudes for massive spinning particles
- Simplest massive spin *S* interacting with photon:

Spinors associated with momenta of particles 1 and 2

$$\mathcal{A}_+ = \sqrt{2} i \, em \, x \langle \mathbf{12} \rangle^{2S}$$

$$\mathcal{A}_- = \sqrt{2} i \, em \, \frac{1}{x} [\mathbf{12}]^{2S}$$

Spinors associated with momenta of particles 1 and 2



$$x = \frac{1}{m} \epsilon_+ \cdot (p_1 - p_2)$$

Large spins

In the classical region with a very large spin, the spinor product simplifies:

$$\langle \mathbf{12} \rangle^{2S} \simeq \left(1 + \frac{1}{2S}\frac{s \cdot \bar{q}}{m}\right)^{2S} \to \exp(a \cdot \bar{q})$$

Vector a^μ parameterises spin, has dimension of length

Electrodynamic amplitudes for particles with classical spin are

$$\mathcal{A}_+ = \sqrt{2} i\, em\, x e^{a \cdot \bar{q}}, \qquad \mathcal{A}_- = \sqrt{2} i\, em\, \frac{1}{x} e^{-a \cdot \bar{q}}$$

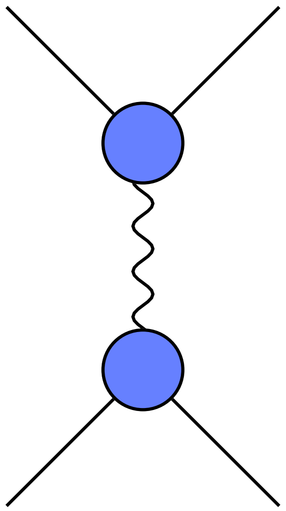
Guevara, Ochirov, Vines

Arkani-Hamed, Huang, DOC

Large spins

- Fuse three point amplitudes to get four point (BCFW)

$$\mathcal{A}_4 \sim \frac{1}{q^2}\left(\mathcal{A}_+^{\rm up}\mathcal{A}_-^{\rm dn} + \mathcal{A}_-^{\rm up}\mathcal{A}_+^{\rm dn}\right)$$



$a = a_1 + a_2$

Levi-Civita

$$\mathcal{A}_4 = \frac{1}{q^2} 2 e_1 e_2 m_1 m_2 \left(\frac{x_u}{x_d} e^{a\cdot q} + \frac{x_d}{x_u} e^{-a\cdot q}\right)$$

$$\frac{x_u}{x_d} = u_1 \cdot u_4 - i\frac{\epsilon(q, u_1, a, u_4)}{a\cdot q} = \cosh w - i\frac{\epsilon(q, u_1, a, u_4)}{a\cdot q}$$

u_i are proper velocities

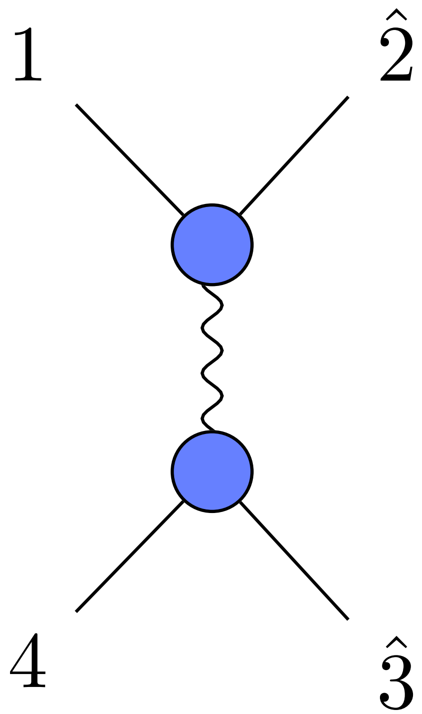
w is the associated rapidity

Large spins

- But we want to compare to Kerr impulse: double copy to gravity

$$\mathcal{M}_{++} = i\,\frac{\kappa}{2} m^2\, x^2 e^{a\cdot\bar{q}}, \qquad \mathcal{M}_{--} = i\,\frac{\kappa}{2} m^2\, \frac{1}{x^2} e^{-a\cdot\bar{q}}$$

- BCFW



$$\frac{x_u^2}{x_d^2} = \left(u_1\cdot u_4 - i\,\frac{\epsilon(q,u_1,a,u_4)}{a\cdot q}\right)^2$$

$$= \cosh^2 w - 2i\cosh w\,\frac{\epsilon(q,u_1,a,u_4)}{a\cdot q} + \sinh^2 w + \mathcal{O}(q^2)$$

Gram determinant

Cancels propagator: irrelevant contact term

Large spins

- Hence four point amplitude (in classical region)

$$\mathcal{M}(q) = -\frac{\kappa^2 m_1^2 m_2^2}{4q^2}\left[\left(\cosh 2w + 2i\cosh w\frac{\epsilon(u_1,u_2,a,q)}{a\cdot q}\right)e^{-q\cdot a} + \left(\cosh 2w - 2i\cosh w\frac{\epsilon(u_1,u_2,a,q)}{a\cdot q}\right)e^{q\cdot a}\right]$$

- Impulse involves $\hat{\delta}(q\cdot u_1)\hat{\delta}(q\cdot u_2)q^\mu\mathcal{M}(q)$ … leads to simplification

$$q^\mu = A\epsilon^\mu(q,u_1,u_2) + B\epsilon^\mu(u_1,u_2,a) + C\epsilon^\mu(u_2,a,q) + D\epsilon^\mu(a,q,u_1)$$

$$\therefore\ \epsilon(a,q,u_1,u_2)\,q^\mu = a\cdot q\,\epsilon^\mu(q,u_1,u_2) + \mathcal{O}(q^2)$$

Large spins

- Then just algebra to see impulse is

$$\Delta p_1^\mu = G m_1 m_2 \mathrm{Re} \int \hat{d}^4\bar{q}\; \hat{\delta}(\bar{q}\cdot u_1)\hat{\delta}(\bar{q}\cdot u_2)\frac{ie^{-i\bar{q}\cdot(b+ia)}}{\bar{q}^2}$$
$$\times\left(\bar{q}^\mu \cosh 2w + 2i\cosh w\, \epsilon^\mu(\bar{q}, u_1, u_2)\right)$$

Vines

- Thing to note: spin only appears as additive shift

$$b \to b + ia$$

  - Simple consequence of exponentiation in three-point amplitude
  - Same is true in electrodynamic case of $\sqrt{\text{Kerr}}$

Large spins

- Newman-Janis transformation: upgrade Schwarzschild to Kerr by replacing z with $z + ia$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = \frac{2M}{r} k_\mu k_\nu \to h_{\mu\nu}^{\text{Kerr}}$$

all orders (Newman-Janis)

$$A_\mu = \frac{e}{r} k_\mu \qquad \to A_\mu^{\sqrt{\text{Kerr}}}$$

Lynden-Bell
Monteiro, White, DOC

- Classical double copy checks out for Kerr

Magnetic Amplitudes

Magnetic amplitudes

Caron-Huot and Zahraee

- Consider a static dyon at the origin
  - Electric charge density $\rho = q\delta^3(x)$
  - Magnetic charge density $\rho_m = g\delta^3(x)$
- Can perform an electric-magnetic duality rotation to remove magnetic charge:

$$\underline{E}' = \underline{E}\cos\theta + \underline{B}\sin\theta$$
$$\underline{B}' = -\underline{E}\sin\theta + \underline{B}\cos\theta$$

$$\rho'_m = -\rho\sin\theta + \rho_m\cos\theta = 0$$

- Or, given pure electric charge, rotate to induce magnetic charge

Magnetic amplitudes

- Covariantly, the duality rotation is

$$F_{\mu\nu} = F_{\mu\nu}\cos\theta + F^*_{\mu\nu}\sin\theta$$
$$F^*_{\mu\nu} = -F_{\mu\nu}\sin\theta + F^*_{\mu\nu}\cos\theta$$

$$F^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$$

- Or in terms of self / anti-self dual parts

$$F'_{\mu\nu} - iF'^*_{\mu\nu} = e^{i\theta}(F_{\mu\nu} - iF^*_{\mu\nu})$$
$$F'_{\mu\nu} + iF'^*_{\mu\nu} = e^{-i\theta}(F_{\mu\nu} + iF^*_{\mu\nu})$$

Magnetic amplitudes

- Take pure electric three point amplitudes

$$\mathcal{A}_+ = \sqrt{2} i\, em, \qquad \mathcal{A}_- = \sqrt{2} i\, em\, \frac{1}{x}$$

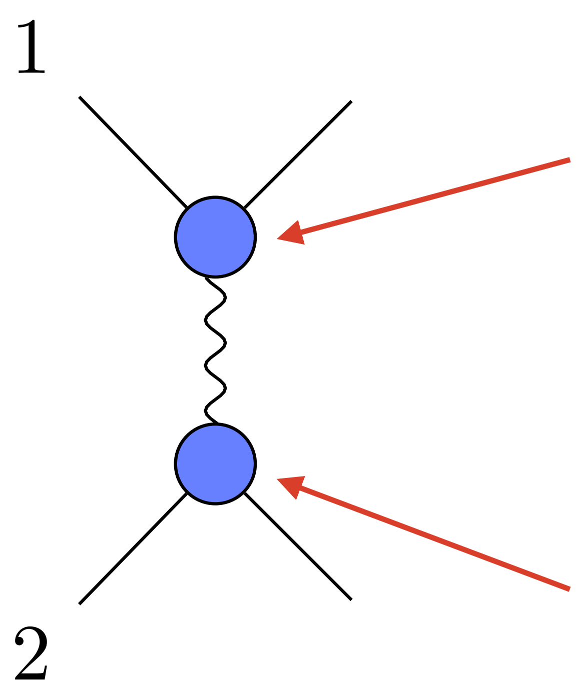
- These correspond to self- / anti-self-dual fields
- Thus duality rotation

$$\mathcal{A}_+ = \sqrt{2} i\, emx e^{i\theta}, \qquad \mathcal{A}_- = \sqrt{2} i\, em\, \frac{1}{x} e^{-i\theta}$$

- Angle only becomes physical if it corresponds to a misalignment with other particles

Magnetic amplitudes

- For example, scatter two dyons *Huang, Kol, DOC*
- 4 point amplitude: BCFW again



$$\mathcal{A}_+ = \sqrt{2}i\, e_1 m_1 x_u e^{i\theta_1}, \ \mathcal{A}_- = \sqrt{2}i\, e_1 m_1 \frac{1}{x_u} e^{-i\theta_1}$$

$$\mathcal{A}_+ = \sqrt{2}i\, e_2 m_2 x_d\, e^{i\theta_2}, \ \mathcal{A}_- = \sqrt{2}i\, e_2 m_2 \frac{1}{x_d} e^{-i\theta_2}$$

- Result depends only on misalignment $\theta = \theta_1 - \theta_2$

Magnetic amplitudes

- Very similar to Kerr case, 4 pt amplitude is

Spurious reference vector

$$\mathcal{A}(q) = 2e_1e_2m_1m_2\frac{1}{q^2}\left[\left(\cosh w - i\frac{\epsilon(\eta, q, u_1, u_2)}{\eta\cdot q}\right)e^{i\theta} + \left(\cosh w + i\frac{\epsilon(\eta, q, u_1, u_2)}{\eta\cdot q}\right)e^{-i\theta}\right]$$

- The unphysical vector drops out in the impulse, same mechanism as Kerr

$$\epsilon(\eta, q, u_1, u_2)\, q^\mu = \eta\cdot q\,\epsilon^\mu(q, u_1, u_2) + \mathcal{O}(q^2)$$

- Final result agrees with direct classical calculation

Magnetic amplitudes

- Classical double copy: dyon → Taub-NUT
  - Taub-NUT has a mass and "NUT" charge

$$ds^2 = f(r)(dt + 2n\cos\theta d\phi)^2 - f(r)^{-1}dr^2 - (r^2 + n^2)d\Omega^2$$

$$f(r) = \frac{(r - r_+)(r - r_-)}{r^2 + n^2}, \quad r_\pm = m \pm r_0, \quad r_0^2 = m^2 + n^2$$

  - Recover Schwarzschild (in Schwarzschild coords) when $n = 0$:

$$f(r) \to \frac{(r - 2m)r}{r^2}$$

Magnetic amplitudes

- Amplitudes for probe scattering off Taub-NUT by the double copy

$$\mathcal{M}_{++} = i\,\frac{\kappa m^2}{2} x^2 e^{2i\theta}, \qquad \mathcal{M}_{--} = i\,\frac{\kappa m^2}{2}\,\frac{1}{x^2} e^{-2i\theta}$$

$$\Rightarrow \mathcal{M}(q) = -\frac{\kappa^2 m_1^2 m_2^2}{4q^2}\left[\left(\cosh 2w + 2i\cosh w\,\frac{\epsilon(u_1,u_2,\eta,q)}{\eta\cdot q}\right) e^{2i\theta} + \left(\cosh 2w - 2i\cosh w\,\frac{\epsilon(u_1,u_2,\eta,q)}{\eta\cdot q}\right) e^{-2i\theta}\right]$$

Huang, Kol, DOC

- Spurious vector drops out in impulse
    - Complete agreement with direct calculation in Taub-NUT

Conclusions

Classical double copy present. Dialog between amplitudes & GR:

- Double copy: simplicity in perturbative gravity
- Known exact solutions: exotic amplitudes

- So what about rest of Plebanski-Demianski family?

- Observables including radiation known to double copy

Kosower, Maybee, DOC

- Analytic continuation between scattering and bound cases?

Kälin and Porto

Golden age for gravity