Non-perturbative Results for Supersymmetric Yang-Mills Theory and Supersymmetry on the Lattice

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1. Introduction: motivations and obstacles for SUSY on the lattice
2. Simulations of $\mathcal{N} = 1$ supersymmetric Yang-Mills theory
3. Towards general simulations of supersymmetric gauge theories in four dimensions

Prospects of supersymmetry on the lattice

Non-perturbative physics from first principles:

1. SUSY BSM physics: non-perturbative breaking scenarios
2. Lessons from SUSY theories for a general understanding of strong interactions
3. Gauge $\leftrightarrow$ Gravity duality:
    - $\leftarrow$ Predictions to be verified and extended with numerical methods.
    - $\rightarrow$ Insights into quantum gravity from SUSY gauge theories.

SUSY breaking and the Leibniz rule on the lattice

No-Go theorem: locality contradicts with SUSY

There is no Leibniz rule for a discrete derivative operator. The action can only be invariant with a non-local derivative and non-local product rule. [GB],[Kato,Sakamoto,So],[Nicolai,Dondi]

Further problems [G.B., S. Catterall, arXiv:1603.04478]:

- fermonic doubling problem, Wilson mass term
- gauge fields represented as link variables

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"The lattice is the only valid non-perturbative definition of a QFT and it can not be combined with SUSY. Therefore SUSY can not exist!" (Lattice theorist)

Comparison to other symmetries on the lattice

Chiral Symmetry

- Nielsen-Ninomiya theorem:
  locality contradicts with chiral symmetry
- Ginsparg-Wilson relation: $\{\gamma_5, D\} = 2aD\gamma_5 D$
- fine-tuning

Space-time symmetries

- subgroup of symmetry preserved on the lattice
- ensures irrelevance of symmetry breaking operators

General solution by generalized Ginsparg-Wilson relation?

"Mrs. RG, the good physics teacher..."
(Peter Hasenfratz)

Symmetry in the continuum ($S[(1+\varepsilon\tilde{M})\varphi] = S[\varphi]$) implies relation for lattice action S_L:

Generalized Ginsparg-Wilson relation

$$M^{ij}_{nm}\phi^j_m \frac{\delta S_L}{\delta \phi^i_n} = (M\alpha^{-1})^{ij}_{nm}\left(\frac{\delta S_L}{\delta \phi^j_m}\frac{\delta S_L}{\delta \phi^i_n} - \frac{\delta^2 S_L}{\delta \phi^j_m \delta \phi^i_n}\right)$$

$$\Phi[\tilde{M}\varphi] = M_{nm}\Phi_m[\varphi]$$

Still open problem how to find solutions. [GB, Bruckmann, Pawlowski]

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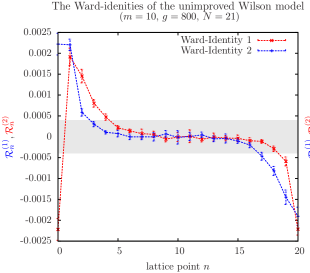
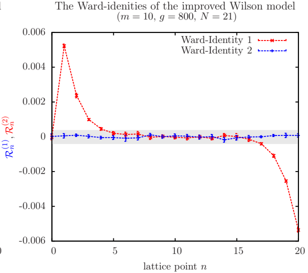
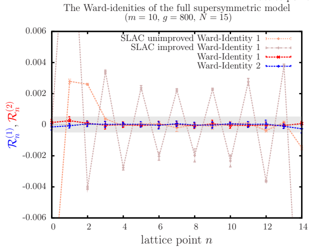
...but we still don't completely understand her lesson.

Sketch of solutions







- only model dependent solutions
- partial realization of extended SUSY
- non-local actions
- otherwise: fine tuning.

Super Yang-Mills theory

Supersymmetric Yang-Mills theory:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\lambda}\not{D}\lambda - \frac{m_g}{2}\bar{\lambda}\lambda\right]$$

- supersymmetric counterpart of Yang-Mills theory; but in several respects similar to QCD
- λ Majorana fermion in the adjoint representation
- SUSY transformations: $\delta A_\mu = -2i\bar{\lambda}\gamma_\mu\varepsilon$, $\delta\lambda = -\sigma_{\mu\nu}F_{\mu\nu}\varepsilon$

Why study supersymmetric Yang-Mills theory on the lattice ?

1. extension of the standard model
   - gauge part of SUSY models
   - understand non-perturbative sector: check effective actions etc.
2. controlled confinement [Ünsal,Yaffe, Poppitz] :
   - compactified SYM: continuity expected
   - small R regime: semiclassical confinement
3. connection to QCD [Armoni,Shifman]:
   - orientifold planar equivalence: SYM $\leftrightarrow$ QCD
   - Remnants of SYM in QCD ?
   - comparison with one flavor QCD

Supersymmetric Yang-Mills theory: Symmetries

SUSY

- gluino mass term m_g $\Rightarrow$ soft SUSY breaking

$U_R(1)$ symmetry, "chiral symmetry": $\lambda \to e^{-i\theta\gamma_5}\lambda$

- $U_R(1)$ anomaly: $\theta = \frac{k\pi}{N_c}$, $U_R(1) \to \mathbb{Z}_{2N_c}$
- $U_R(1)$ spontaneous breaking: $\mathbb{Z}_{2N_c} \overset{\langle\bar{\lambda}\lambda\rangle \neq 0}{\longrightarrow} \mathbb{Z}_2$

Supersymmetric Yang-Mills theory on the lattice

Lattice action:

$$\mathcal{S}_L = \beta \sum_P \left(1 - \frac{1}{N_c}\Re U_P\right) + \frac{1}{2}\sum_{xy} \bar{\lambda}_x \left(\mathrm{D}_w(m_g)\right)_{xy} \lambda_y$$

- Wilson fermions:

$$\mathrm{D}_w = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu)_{\alpha,\beta} T_\mu + (1+\gamma_\mu)_{\alpha,\beta} T_\mu^\dagger\right] + \text{clover}$$

  gauge invariant transport: $T_\mu \lambda(x) = V_\mu \lambda(x+\hat{\mu})$;

$$\kappa = \frac{1}{2(m_g+4)}$$

- links in adjoint representation: $(V_\mu)_{ab} = 2\mathrm{Tr}[U_\mu^\dagger T^a U_\mu T^b]$ of SU(2), SU(3)

Lattice SYM: Symmetries

Wilson fermions:

- explicit breaking of symmetries: ~~chiral Sym. ($U_R(1)$)~~, ~~SUSY~~

fine tuning:

- add counterterms to action
- tune coefficients to obtain signal of restored symmetry

special case of SYM:

- tuning of m_g enough to recover chiral symmetry [1]
- same tuning enough to recover supersymmetry [2]

[1] [Bochicchio et al., Nucl.Phys.B262 (1985)]

[2] [Veneziano, Curci, Nucl.Phys.B292 (1987)]

Recovering symmetry

Fine-tuning:

chiral limit $=$ SUSY limit $+O(a)$, obtained at critical $\kappa(m_g)$

- no fine tuning with Ginsparg-Wilson fermions (overlap/domainwall) fermions[3]; but too expensive

practical determination of critical κ:

- limit of zero mass of adjoint pion $(a-\pi)$

$\Rightarrow$ definition of gluino mass: $\propto (m_{a-\pi})^2$

- cross checked with SUSY Ward identities

[3] [Fleming, Kogut, Vranas, Phys. Rev. D 64 (2001)], [Endres, Phys. Rev. D 79 (2009)], [JLQCD, PoS Lattice 2011]

Low energy effective theory

	multiplet[1]	**multiplet**[2]
scalar	meson a$-f_0$	glueball 0^{++}
pseudoscalar	meson a$-\eta'$	glueball 0^{-+}
fermion	gluino-glue	gluino-glue

- colourless bound states at low energies consistent with SUSY
- simplest assumption: chiral multiplet
- glueballs, gluino-glueballs, gluinoballs (mesons)

[1] [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]

[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

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Supersymmetry

Particles must have same mass.

- colourless bound states at low energies consistent with SUSY
- simplest assumption: chiral multiplet
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Lattice bound dates

	multiplet[1]	multiplet[2]
scalar	meson a–f_0: $\bar{\lambda}\lambda$	glueball 0^{++}
pseudoscalar	meson a–η': $\bar{\lambda}\gamma_5\lambda$	glueball 0^{-+}
fermion	gluino-glue: $\sigma_{\mu\nu}\mathrm{tr}\left[F^{\mu\nu}\lambda\right]$	gluino-glue

Challenging to get signal:

- flavour singlet meson states
- glueballs
- gluino-glue spin-1/2 state
- mass determination from exponential correlator
- mixing of multiplets considered

History and status of the project

SU(2) SYM:

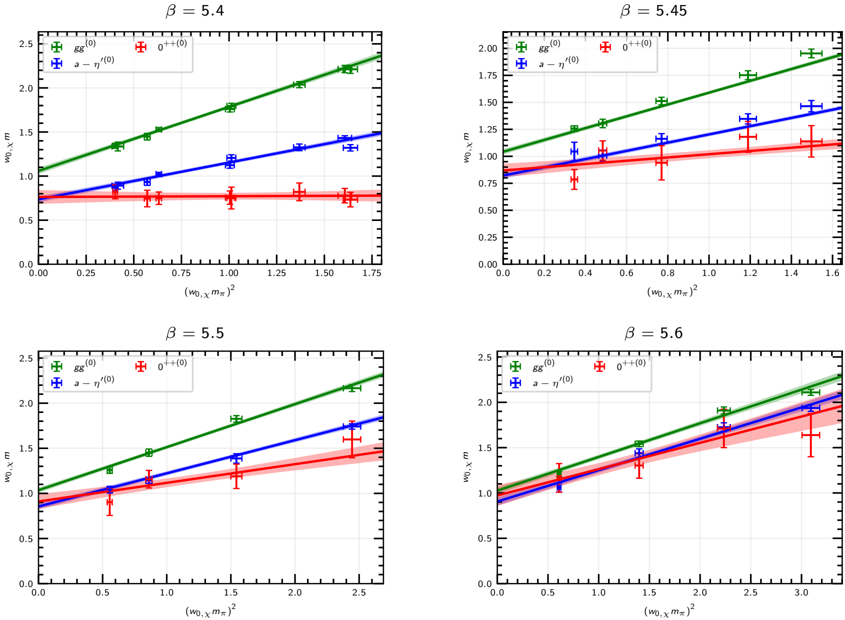
- multiplet formation found in the continuum limit of SU(2) SYM [JHEP 1603, 080 (2016)]

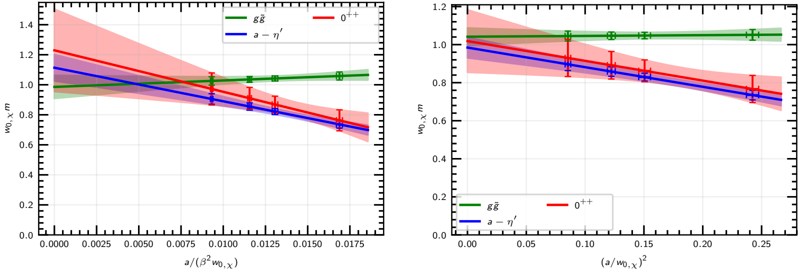
SU(3) SYM:

- adjoint representation much more demanding than fundamental one (limited to small lattice sizes)
- first SU(3) simulations [LATTICE99,LATTICE2016,LATTICE2017]
- results presented here: [S. Ali, GB, H. Gerber, I. Montvay, G. Münster,S. Piemonte, P. Scior PRL (2019)]

Advanced methods of lattice QCD required:

- disconnected contributions [LATTICE2011]
- eigenvalue measurements [GB,Wuilloud, Comput. Phys. Commun. 183 (2012)]
- mixing using variational methods [JHEP 04 (2019)]


β = 5.4
β = 5.45
β = 5.5
β = 5.6
gg(0)
0++(0)
a − η′(0)
w0,χ m
(w0,χ mπ)2



Fit	$w_0 m_{g\tilde{g}}$	$w_0 m_{0^{++}}$	$w_0 m_{\mathrm{a}-\eta'}$
linear fit	0.917(91)	1.15(30)	1.05(10)
quadratic fit	0.991(55)	0.97(18)	0.950(63)
SU(2) SYM	0.93(6)	1.3(2)	0.98(6)

Confirmed also by Ward identities.

([Eur.Phys.J. C78 (2018) no.5, 404])

Phase transitions in supersymmetric Yang-Mills theory

In QCD:

- center symmetry breaking
- chiral symmetry breaking

$\rightarrow$ both transitions crossover

In SYM:

- center symmetry
- chiral symmetry (at $m_g = 0$)

$\rightarrow$ two independent transitions

Deconfinement:

- $T > T_c^{\text{deconf.}}$ plasma of gluons and gluinos
- order parameter: Polyakov loop (P_L)

Chiral phase transitions:

- $T > T_c^{\text{chiral}}$ fermion condensate melts, chiral symmetry restored
- order parameter: $\langle \bar{\lambda}\lambda \rangle$

Expectations for the phase transitions

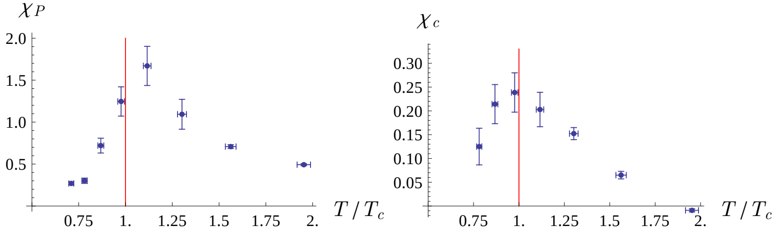
Analytic predictions:

- t'Hooft anomaly matching: $T_c^{\text{deconf.}} \leq T_c^{\text{chiral}}$
- string theory conjectures for SYM: $T_c^{\text{deconf.}} = T_c^{\text{chiral}}$

Earlier lattice investigations with 2 adjoint Dirac fermions (SU(3)):

- $T_c^{\text{chiral}} \approx 175(50)\, T_c^{\text{deconf.}}$ [Kogut, Polonyi, Wyld, Sinclair, Phys. Rev. Lett. **54** (1985)]
- $T_c^{\text{chiral}} \approx 7.7(2.1)\, T_c^{\text{deconf.}}$ [Karsch,Lutgemeier,Nucl.Phys.B 550 (1999)]
- $T_c^{\text{chiral}} \approx 7.8(2)\, T_c^{\text{deconf.}}$ [Engels, Holtmann, Schulze, Nucl. Phys. B **724** (2005)]
- systematics not under control, most likely conformal theory

SU(2) SYM at finite temperature



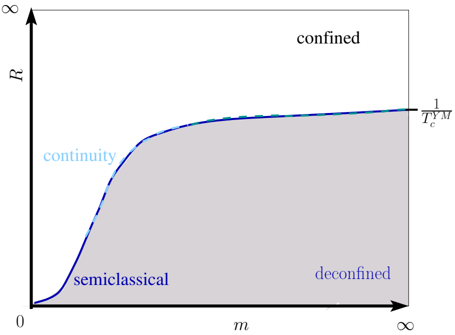
- coincidence of deconfinement and chiral transition $T_c^{\mathrm{chiral}} = T_c^{\mathrm{deconf.}}$ (within current precision)

  [JHEP 1411 (2014) 049], [GB, López, Piemonte, Phys. Rev. D **100** (2019)]

- second order deconfinement transition

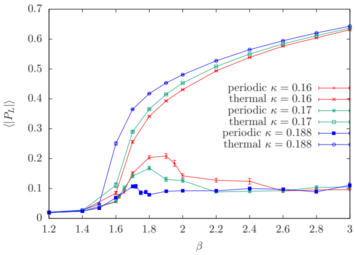
$$\frac{T_c(\mathrm{SYM})}{T_c(\text{pure Yang-Mills})} = 0.826(18).$$

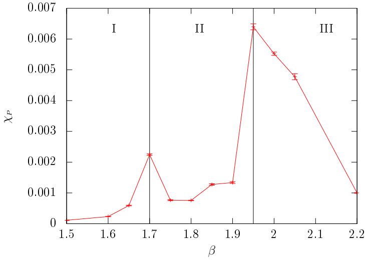
Turn off deconfinement by change of boundary conditions



- thermal $\rightarrow$ periodic fermion boundary conditions
  $Z(\beta_B = \frac{1}{T})$ (thermal ensemble) $\rightarrow \tilde{Z}(\beta_B = R)$ (Witten index)
- β_B independent $\Rightarrow$ continuity in SYM
- related: Eguchi-Kawai reductions, Hosotani mechanism

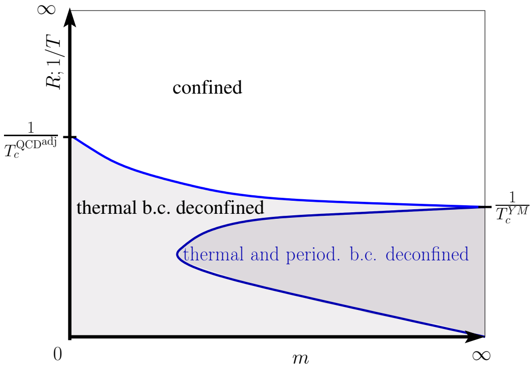
Compactified SYM with periodic boundary conditions





- fermion boundary conditions: thermal $\rightarrow$ periodic
- at small m (large κ) no signal of deconfinement
- intermediate masses: two phase transitions (deconfinement + reconfinement) [GB,Piemonte],[GB, Piemonte, Ünsal]

Turn off deconfinement by change of boundary conditions



- at small radius: stronger confining effect introduced by the lattice fermions

Towards other 4D supersymmetric gauge theories

Generalized tuning approach:

- $O(a)$ SUSY breaking on the lattice
- radiative corrections lead to relevant breaking, compensated by counterterms $\to$ tuning
- $\mathcal{N} = 1$ SYM: only tuning of gluino mass term required, no tuning for Ginsparg-Wilson fermions.

$\Rightarrow$ provide a more general approach for 4D SUSY gauge theories

$\mathcal{N} = 1$ SYM and mixed representations: supersymmetric QCD

- add $N_c \oplus \bar{N}_c$ chiral matter superfield (ψ quarks, Φ_i squarks) to supersymmetric Yang-Mills theory

$$
\begin{aligned}
\mathcal{L}_{SQCD} =& \mathcal{L}_{SYM} + |D_\mu \Phi_1|^2 + |D_\mu \Phi_2^\dagger|^2 + \bar{\psi}(\gamma_\mu D_f^\mu + m)\psi \\
&+ m^2|\Phi_1|^2 + m^2|\Phi_2|^2 \\
&+ i\sqrt{2}g\bar{\lambda}^a \left(\Phi_1^\dagger P_+ + \Phi_2 P_-\right) T^a\psi \\
&- i\sqrt{2}g\bar{\psi}T^a \left(P_-\Phi_1 + P_+\Phi_2^\dagger\right)\lambda^a \\
&+ \frac{g^2}{2}\left(\Phi_1^\dagger T^a\Phi_1 - \Phi_2 T^a \Phi_2^\dagger\right)^2
\end{aligned}
$$

Why consider SQCD

- natural extension of supersymmetric Yang-Mills theory
- relation to possible extensions of the standard model

SQCD analysis of Seiberg et al.:

- $N_f < N_c$ No vacuum
- $N_f = N_c$ confinement and chiral symmetry breaking
- $\frac{3}{2}N_c < N_f < 3N_c$ infrared fixed point (duality)

Like other SUSY theories beyond $\mathcal{N} = 1$ SYM: conformal or near conformal behaviour

Simulations towards SQCD

- large number of tuning parameters, reduced to 6 by Ginsparg-Wilson fermions
- complex Pfaffian, but cancellations due to Pf $\rightarrow$ Pf*
- not well behaved chiral limit:
  - either near conformal (larger $N_f > \frac{3}{2} N_c$)
  - or unstable vacuum (smaller $N_f < N_c$)
  - interesting: $N_f = N_c$ and $N_f = N_c + 1$

Work in progress:

- investigated mixed adjoint-fundamental representation theory
- tested simulation program with Yukawa couplings [arXiv:1811.01797 [hep-lat]]
- perturbative tuning [Costa, Panagopoulos], [Wellegehausen, Wipf], [Giedt], [Curci, Veneziano]

From $\mathcal{N} = 1$ to $\mathcal{N} = 4$ supersymmetric Yang-Mills theory

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory is obtained from $\mathcal{N} = 1$ supersymmetric Yang-Mills theory in 10 dimensions via dimensional reduction.

- 1 Majorana-Weyl fermion $\rightarrow$ 4 Majorana fermions
- 6 additional gauge fields become scalars X_i
- Yukawa interactions

Additional bosonic term:

$$S_{\mathrm{B}} = \int d^4x \, \left[\frac{1}{2} D_\mu X^i D^\mu X^i + \frac{1}{4} [X^i, X^j]^2 \right]$$

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory

Interesting theory:

- gauge-gravity duality, string theory...

Large supersymmetry reduces fine tuning:

- naive expectation: large SUSY, large fine tuning
- However: maximal SUSY allows to preserve sub-group on the lattice
- preserved SUSY constrains counterterms

Twisted formulation: example $\mathcal{N} = (2, 2)$ SYM in two dimensions

Field content:

- 2 Majorana fermions λ^I
- two scalar fields B^I, and two gauge fields A_i

Twisted symmetry group:

- $SO(2)_E$ Lorentz group, $SO(2)_I$ flavour symmetry
- decompose fields according to $SO(2)'$=diag($SO(2)_E \times SO(2)_I$)

Q becomes a matrix:

$$Q = qI + q_\mu \gamma_\mu + q_{12}\gamma_1\gamma_2$$

Scalar supercharge $\{q, q\} = 0$: q can be preserved on the lattice

Example $\mathcal{N} = (2,2)$ SYM in two dimensions

- action is a q-exact form
- scalar fields transform as vectors and are combined with A into complexified gauge field
- Dirac-Kähler fermions $(\eta, \psi_\mu, \chi_{12})$

Lattice structure:

- ψ_μ, A_μ on links, χ_{12} on (backward) diagonal

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory on the lattice

Similar construction [Ünsal, Kaplan]:

- $\mathrm{SO}(4)_E$ Lorentz group, SU(4) R-symmetry contains $\mathrm{SO}(4)_R \times \mathrm{U}(1)$ part
- choose diagonal $\mathrm{SO}(4)'$ part
- 5 complex "gauge" fields
- 16 fermionic degrees of freedom $(\eta, \psi_a, \chi_{ab})$
- lattice structure with 5 basis vectors

$\Rightarrow$ reduced fine tuning [Catterall, Dzienkowski, Giedt, Joseph, Wells, JHEP **04** (2011)]

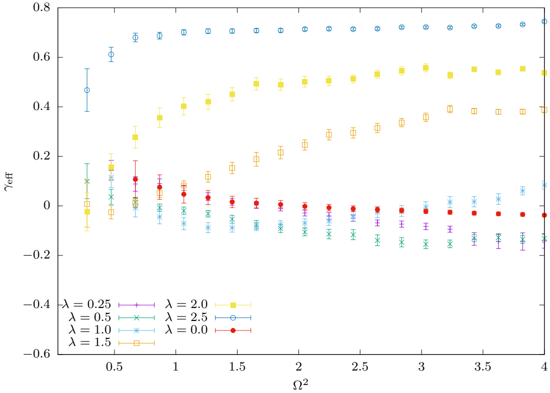
Continuum limit of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory

Non-trivial continuum limit:

- IR supersymmetry restoration
- conformal invariance in continuum limit, broken by lattice artefacts
- β-function vanishes, mass anomalous dimension vanishes
- estimate relevant scales by measuring RG flow
- effective RG flow for mass anomalous dimension mode number from Dirac operator eigenvalue spectrum

Mass anomalous dimension in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on the lattice

Obtained from the mode number (preliminary)
(integrated spectral density of $D^\dagger D$)



$\mathcal{N} = 4$ supersymmetric Yang-Mills theory on the lattice

Challenges:

- complex Pfaffian relevant at larger couplings
- flat directions introduced by scalar fields
- stabilizing the simulations [Catterall, Giedt, Jha (2018)]
- continuum limit at larger couplings

Conclusions

- simulation of supersymmetric theories on the lattice is still in some aspects an open theoretical problem

$\mathcal{N} = 1$ supersymmetric Yang-Mills theory:

- theoretical problem is solvable, practical challenges
- interesting non-perturbative physics like the phase diagram can be investigated on the lattice

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory:

- alternative approach with conserved SUSY
- challenges: larger couplings, stabilization of simulations

Open challenges and ongoing efforts:

- generalizing the tuning approach: Can we simulate SQCD and $\mathcal{N} = 2$ supersymmetric Yang-Mills?